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# Estimation of the mass and luminosity of Polaris from resonance hypothesis

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**Abstract**

The resonance hypothesis for the cause of the decreasing event in the pulsation amplitude of the Cepheid component of Polaris is proposed. If Polaris is at the center of the 2:1 resonance between the first and the fourth overtone modes, we can estimate the mass and luminosity from the results of the linear nonadiabatic pulsation models using resonance conditions. The estimated stellar parameters from the resonance hypothesis are within the possible parameter range.
**Key words:** stars: individual (Polaris) — stars: oscillations — stars: variables: Cepheids

## 1. Introduction

Classical Cepheids are population I yellow supergiants pulsating radially. These stars are well known to be the standard candles for the extragalactic distance scale, thanks to the period-luminosity relation. Their fundamental properties are explained by the stellar pulsation theory, however, it is pointed out that there are some discrepancies between results from the stellar evolution theory and those from the pulsation theory.

Among those objects, Polaris ($P = 3.9696\,$days, F8 I; GCVS: Samus' et al. 2017) is one of the most popular and well-observed object. However, there is still some unsolved problems concerning this object.

Feast and Catchpole (1997) investigated the period-luminosity relation of Cepheids with Hipparcos trigonometric parallax, and pointed out that the weight of Polaris on this relation is much greater than others. They also indicated that Polaris is considered to be the first overtone pulsator. However, Turner (2004) pointed out the existence of the overlooked cluster around Polaris and derived closer distance $94 \pm 4\,$pc (see also Turner et al. 2005), from which they concluded that Polaris pulsates in the fundamental mode. On the contrary, revised Hipparcos value is $132.6^{+2.9}_{-1.9}\,$pc (van Leeuwen 2007). Selected recent distance estimations of Polaris including Gaia DR2 estimate for companion Polaris B ($137.14^{+0.53}_{-0.52}\,$pc) are summarized in Engle, Guinan, and Harmanec (2018).

Because interferometric angular diameter is estimated (Mérand et al. 2006), adapting longer distance means not only brighter luminosity but also the larger radius and prefers the first overtone mode pulsation, and vise versa. From two different sets of luminosity and radius, we can estimate two different mass to obtain the observed period. Therefore, there are still some uncertainty for the stellar parameters for Polaris.

The evolutionary status and crossing number of Polaris is also one of the controversial subjects. The change of the pulsation period with the stellar evolution of the Polaris is summarized in Turner et al. (2005). Saitou (1989) classified Polaris as the first crossing objects, but mentioned that we need further investigation to identify the pulsation mode. Using the shorter distance ($99 \pm 2\,$pc), Turner et al. (2013) concluded that Polaris is pulsating in the fundamental mode and crossing the instability strip for the first time. Neilson (2014) used new stellar evolution models and pointed out that Polaris is in its third crossing and pulsate in the first overtone mode. More recently, Bond et al. (2018) pointed out that isochrones for Polaris A (Cepheid) and its wide companion B are different. In summary, the evolutionary status of Polaris is not clear yet.

The decrease in the pulsation amplitude of Polaris from about 1.5 mag in 1940s to 0.05 mag in 1980s was first reported by Arellano Ferro (1983) using photometric observations and confirmed by Kamper et al. (1984) by radial velocity observations. Fernie et al. (1993) pointed out that the decrease in the amplitude is exponential and deduced that the pulsation will stop in 1994. However, Bruntt et al. (2008) reported that pulsation amplitude increased again. More recent observations indicate the further increase (Mkrtichian et al. 2014) and the following possible decrease (Usenko et al. 2018). It is also worth noting that more gradual brightening in longer time scale, say over 2000 years is also pointed out (Engle et al. 2004).

Since the observed decrease in the pulsation amplitude is remarkable, some authors have proposed a possible cause of this behavior. Moskalik and Ogłoza (2000) speculated that the observed decrease is a part of the periodic or quasi-periodic amplitude modulation near the 2:1 resonance between the first and the fourth overtone radial modes. Stothers (2009) pointed out that if the amplitude modulation is cyclic, it resembles the Blazhko effect observed in RR Lyrae stars. He proposed a magnetoconvective cycle in the stellar envelope as a possible explanation. However, it is still unclear whether the decrease is a cyclic or intrinsically one-time event. Both of the above two proposals are assuming cyclic variation, and we seem to have no explanation if the event is one-time.

In this paper, we would like to propose the resonance itself as the possible cause of the decrease of the pulsation amplitude which is in one-time only in their life as a classical Cepheid. Furthermore, the mass and the luminosity under the resonance hypothesis are estimated from the linear nonadiabatic pulsation calculations and compared with other estimations. In section 2, the resonance hypothesis for the decrease in the pulsation amplitude is explained. Models and their results of the linear nonadiabatic pulsation calculations are shown in section 3. The discussions are presented in section 4. We conclude in section 5.

## 2. Resonance hypothesis

2:1 resonance between two radial pulsation modes is theoretically investigated in relation to the bump Cepheid phenomena between the fundamental and the second overtone modes (Takeuti and Aikawa 1981; Buchler and Kovacs 1986: see also Aikawa 1984). The apparent phenomena between the bump Cepheids and Polaris amplitude differ largely. The decrease of pulsation amplitude of Polaris is observed in one star within a hundred years, while the bump Cepheid phenomena are observed in many stars with period around 10 days. However, we need to consider that the coupling coefficients between related modes have only calculated for the first three modes, namely fundamental, first overtone, and second overtone. It is known that the higher overtones usually have smaller amplitude if the amount of oscillating energy is the same, therefore, it seems probable that the decrease in amplitude occurs in the narrower period range and is deeper around the period for the exact resonance for the fourth overtone case.

From the observational data, the center of the 2:1 resonance for the first and fourth overtone for the light curve data is estimated around 3.2 days from the jump of the Fourier phases $\phi_{21}$. However, for the radial velocity data, it is pointed out that there is no jump in the Fourier phases around the above period and changes smoothly for all the period range. Because the center of the resonance need to occur at the same period for both of the light curve and radial velocity data, Kienzle et al. (1999) estimated the center to occur at the longer period, 4.58 days in our Galaxy. Furthermore, Feuchtinger, Buchler, and Kolláth (2000) surveyed their results obtained by hydrodynamic models, and reproduced the observed change of the Fourier parameters for both the light curve and the radial velocity by their models. They estimated the center of the resonance is near 4.2 days in the case of their models.

For Polaris itself, Moskalik and Ogłoza (2000) analyzed radial velocity data to derive its vicinity to the resonance center. They estimated that the ratio between the fourth and the first overtone periods for Polaris is 0.51. However, considering much larger uncertainty of Polaris than other stars for the estimated Fourier parameter $\phi_{21}$ into account, it seems possible that Polaris is nearer to the resonance center than their estimate.

If the Polaris is at the center of the resonance, we can confine the possible region of its properties from the resonance condition and the observed value of the period. The first overtone also needs to be pulsationally unstable, and the fourth overtone to be stable. In the next section, we would like to present linear nonadiabatic pulsation calculation and its results.

## 3. Linear non-adiabatic pulsation calculations

### *3.1. Models*

We used the same code used in Ishida (1995, 2017), namely, Castor (1971) type procedure with OPAL opacity tables (Iglesias et al. 1992; Rogers & Iglesias 1992) for calculation of the linear nonadiabatic periods. As the purpose of this paper is to examine the possibility that the decrease of the amplitude in Polaris may be explained by the resonance, the envelope model is purely radiative i.e. the effect of the convective energy transport is not included in all of the models. This effect is discussed in section 4.

Results of effective temperature estimations are summarized in Usenko et al. (2005). The mean effective temperatures during the last 60 yrs spread within 5800–6200 K. Therefore, we set the parameter range for effective temperature from 5800 to 6200 K with intervals of 100 K.

Recently, the close companion of the Polaris is resolved by Hubble Space Telescope observation and reported that the dynamical mass of the Cepheid is estimated to be $3.45 \pm 0.75\, M_\odot$ (Evans et al. 2018). Considering Anderson (2018)'s estimation of 7 $M_\odot$ into account, we set also the parameter range widely from 4.0 to 7.5 $M_\odot$ with intervals of 0.1 $M_\odot$.

We can derive luminosity estimation in several ways. We can estimate it from Fouqué et al. (2007)'s period-luminosity relation using the observed period of Polaris, while we need take the possibility into account that the observed period 3.9696 days is for the first overtone. The derived luminosity by this method is about 1150 $L_\odot$ if the observed period corresponds to the fundamental mode while about 1660 $L_\odot$ if the first overtone is assumed. We can also estimate it from the observed mean visual magnitude ($< V >= 1.982$), reddening ($E(B-V) = 0.01 \pm 0.01$), and parallax with Torres (2010)'s polynomial fit for Flower (1996)'s bolometric correction. We can derive several values depending on which parallax we use. For example, we can estimate about 2530 $L_\odot$ for the Gaia DR2 parallax of Polaris B, namely, 7.292 mas. For the preliminary calculations, we set the parameter range from 1000 to 4000 $L_\odot$.

Although it is reported that Polaris has a small supersolar abundance in the majority of elements (Andrievsky et al. 1994), we assume $Z = 0.02$ as the reference composition. Later we would like to discuss the effect of chemical composition.

From the preliminary calculations, we can recognize that the 2:1 resonance seems to be possible only for the first and fourth overtones as the Polaris model, we will show the results only related to these two radial modes. We set the parameter range for the luminosity from 1500 to 4000 $L_\odot$ with intervals of 100 $L_\odot$ hereafter.

The final selected parameter range is summarized in table 1.

### *3.2. Results*

As a typical example of the results, the contour plot for the effective temperature 6000 K is presented in figure 1. The exact resonance with the observed period of Polaris is realized around $M = 5.40\, M_\odot$ and $L = 2470\, L_\odot$.

In the same manner, we can estimate the masses and luminosities for the exact resonance between the first and the fourth overtones with the observed period of Polaris for every $T_{\rm eff}$

**Table 1.** Summary of the model parameters

| Parameter | |
|---|---|
| $T_{\rm eff}$ | 6200–5800 |
| Mass (in $M_\odot$) | 4.0–7.5 |
| $X$ | 0.70 |
| $Z$ | 0.01, 0.02, 0.03 |
| Luminosity (in $L_\odot$) | 1500–4000 |

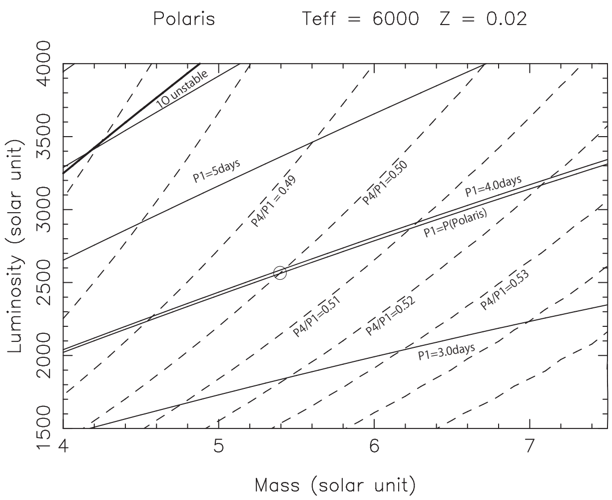


**Fig. 1.** Contour plot for the effective temperature 6000 K. Abscissa is the model mass and ordinate is the model luminosity both in the solar unit. Dashed lines are period ratio contours with corresponding period ratio values of the fourth overtone to the first overtone. Solid lines are the result of the mass-luminosity relations for the first overtone modes corresponding to a given period value. Observed period of the Polaris 3.9696 days is also indicated. The thick line near the top left corner is the instability edge of the first overtone. Below this line, the first overtone is pulsationally unstable. If the exact resonance is realized in the Polaris, the circle point will satisfy the necessary conditions.

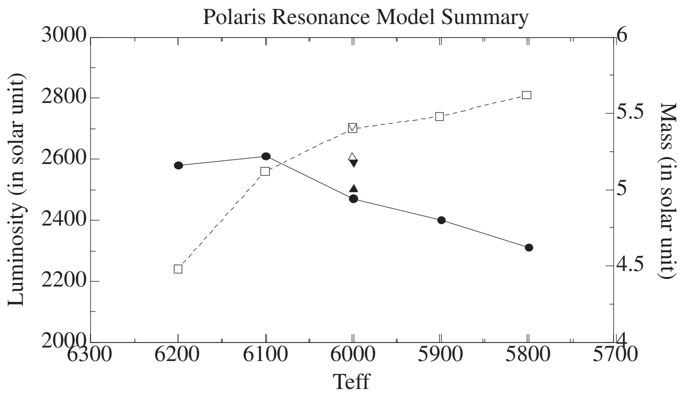


**Fig. 2.** The masses and luminosities derived from the following conditions are plotted for a given $T_{\rm eff}$ value. Period ratio of the fourth to the first overtones is 0.5, namely, the exact 2:1 resonance, the period of the first overtone is the same as the observed period of Polaris, the first overtone is pulsationally unstable, and the forth overtone is pulsationally stable. Filled circles with thick lines are derived luminosities for the $Z = 0.02$ models. Open squares with broken lines are derived masses for the $Z = 0.02$ models. Luminosity scale is given on the left ordinate and mass scale is given on the right ordinate. Mass and luminosity derived for the $Z = 0.01$ models are filled and open upper peaked triangles, respectively. Those for the $Z = 0.03$ models are bottom peaked triangles with the same way.

values. Thus, we can summarize the whole of the results as shown in figure 2. The derived mass gradually increases as we use lower effective temperature, while the derived luminosity mostly decreases.

## 4. Discussions

As described in the previous section, we can derive the mass of Polaris as 5.40 $M_\odot$. As mentioned in the previous section, the dynamical mass of the Cepheid component of the Polaris system is estimated to be $3.45 \pm 0.75\, M_\odot$ (Evans et al. 2018). However, Engle, Guinan, and Harmanec (2018) report as their results for preliminary fit given by evolutionary tracks including rotation effects as $M \sim 6.2$–$6.7\, M_\odot$. The mass for $T_{\rm eff} = 6200$ K models are marginally matched to the dynamical mass estimation. If we adopt $T_{\rm eff} = 6000$ K models, our estimate mass is larger than the dynamical mass, but smaller than the evolutionary mass. The derived mass 5.40 $M_\odot$ seems to be within the possible parameter range.

If we adopt the Gaia DR2 distance to Polaris B as the parallax to the Cepheid component of Polaris, we can estimate the luminosity of the Polaris as 2530 $L_\odot$. It is worth noting that our estimate for the $Z = 0.02$ models with $T_{\rm eff} = 6000$ K is as near to the value from Gaia DR2 as about 2470 $L_\odot$.

$Z = 0.01$, and 0.03 models for $T_{\rm eff} = 6000$ K are also calculated to check the possible effect of $Z$. The derived parameters are also plotted in figure 2. Open marks are for mass estimation and filled marks are for luminosity estimation. We can see that all of the mass estimations are within 0.21 $M_\odot$ width, and luminosity estimations are within 130 $L_\odot$width. The effect of $Z$ seems to be not so large.

For evaluating the effect of convection, we tried to use Radial Stellar Pulsation (RSP) feature of the Modules for Experiments in Stellar Astrophysics (MESA) project, which is described in MESA paper 5 (Paxton et al. 2019). We used almost default values but grid points are increased to be nearly 500, as our no convection model also uses nearly 500 zones. We picked up only the linear results. The most significant difference is that the first overtone modes are pulsationally stable in a wider parameter range than those for no convection models. The stability edge for the first overtone exists at a much shorter than the observed Polaris period. We seem to need more detailed considerations concerning the inclusion of convection.

## 5. Concluding Remarks

We proposed that the 2:1 resonance between the first and the fourth overtone modes itself as the possible origin of the observed pulsation amplitude decrease in the Cepheid component of Polaris. We can estimate the stellar parameters of Polaris if Polaris is indeed in the resonance. From the result of the linear nonadiabatic pulsation calculations, we have shown that the parameters derived from the resonance hypothesis are within the possible parameter range.

Coupling coefficients between some radial pulsation modes are evaluated in some papers (e.g., Takeuti and Aikawa 1981; Klapp et al. 1985). However, their studies are limited to the bump Cepheid phenomena between fundamental and the second overtone modes. We seem to need to evaluate coupling coefficients between the first and fourth overtone modes.

One need to remark that the proposed origin will induce the one-time event during life as a Cepheid. If the amplitude shows clear cyclic change, the modulation proposed by Moskalik and Ogłoza (2000) may be the origin. We need more observations to confirm the possible increase/decrease of the pulsation amplitude.

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