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A new simple method to observe Doppler shift of the meteor head echo using low-cost and low-power ham radio devices and free FFT software

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Abstract

A meteor is a phenomenon in which extraterrestrial material, generally known as a meteoroid, enters the Earth's atmosphere, ionizing the surrounding components and/or emitting light. The surface of a meteoroid moving at high speed reflects very high frequency (VHF) radio waves by melting itself and ionizing the components of the surrounding atmosphere. The radio wave reflected from the head of a meteor is called a meteor head echo and results in a Doppler shift.

Since the early 2000s, some researchers have made multi-static measurements of head echoes using high-power large-aperture pulse radars. Meanwhile, the authors have attempted to observe meteor head echoes with Doppler shifts using traditional low-cost and low-power ham radio devices for over a year. Moreover, by setting the transmitter and the receiver in close proximity, it became clear that the Doppler shift shows a value that follows the meteor velocity component in the line-of-sight direction. This implies that any observer who is involved in amateur radio operations can make observations similar to those obtained from a large-scale pulse meteor radar.

Using this technique, the authors recorded two remarkable Eta Aquarids meteor head echoes in a location very close to the transmitting station, in May 2021. The Doppler shift calculated from the meteor head echo agreed well with the data obtained from the position of the meteor's actual path using the known physical parameters of the Eta Aquarids meteors. We propose that this method facilitates the observation of meteor head echoes at multiple points simultaneously, enabling the estimation of the position and ground velocity of a meteor. This system will help us find the path of a meteorite during daytime hours or on cloudy days.

Key words: meteor head echo — Doppler shift: ham radio devices — Eta Aquarids meteor

1. Introduction

When an extraterrestrial dust particle distributed around the orbit of the Earth is caught by the gravity of the Earth, it enters the Earth's atmosphere and melts and evaporates owing to its own kinetic energy. Subsequently, the dust particles and the surrounding atmospheric components become ionized and/or they start emitting light. In general, this ionization and light emission phenomenon is called a meteor. The body of a meteor is called a meteoroid and is composed of fine dust of extraterrestrial origin, in particular comet or asteroid origin. The ionization takes place approximately in the E region of the ionosphere (80–140 km height). The E region ionized by meteors can be divided into an ionized trail and a localized ionized region surrounding the meteoroid (Kaiser 1953). The ionized trail of a meteor is cylindrical in shape, with dimensions of kilometers in length and meters in diameter. The duration of a meteor trail varies from less than 1 second to several minutes (Sugar 1964). Meteor trails are mostly stationary, except for motion due to atmospheric winds. Appleton and Barnett (1925) discovered that ionized meteor trails reflect less than very high frequency (VHF) radio waves (30–300 MHz) due to the increase in the density of free electrons.

Following this discovery, the study of meteors using the radio method was developed before and after the Second World War (Millman & McKinley 1949). In recent years, meteor trails observed in the E region of the ionosphere have been analyzed in detail, and research on the upper atmosphere has been conducted (Nakamura et al. 1991). In the 1970s, radio observations of meteors using FM broadcasting radio waves in the 76–90 MHz frequency band was initiated in Japan (Suzuki et al. 1976). Subsequently, radio observations of meteors using beacon waves in the VHF (ham radio) band have been conducted (Maegawa 1999). Amateur radio meteor observations using beacon waves have been conducted throughout Japan, and it has become possible to detect the diurnal and annual variations of meteor echoes (Okamoto & Maegawa 2008).

It has been shown that not only the meteor trail but also the meteoroid itself, which enters the Earth's atmosphere at high speed, is heated through collisions with the Earth's atmospheric components. During melting and evaporation, the components of the meteoroid head are converted into plasma and begin to reflect VHF radio waves (Pellinen-Wannberg 2005). VHF radio waves reflected from a localized ionized region surrounding the meteoroid are called the head echo of a meteor, as opposed to the trail echo that originates from the ionization of the upper atmospheric components (Close & Hunt 2000). Because the ionized material surrounding a meteoroid moves at high speed, the reflected radio wave of a meteor head exhibits a Doppler effect owing to the velocity of the moving meteoroid.

The Doppler shift is higher than the original frequency of a wave source when the source approaches an observer and lower when the source moves far away. The same is true when a moving body reflects radio waves. The frequency of radio waves impinging on an approaching object increases, and when the object moves away, the frequency decreases. If a VHF radio wave is directed at a meteoroid that has entered the upper atmosphere at high speed, the ionized components and free electrons become enhanced near the meteoroid. Therefore, due to the Doppler effect, the frequency of the reflected radio wave becomes higher or lower depending on the transmitted radio wave.

Since the early 2000s, some researchers have made multistatic measurements of head echoes using high-power large-aperture pulse radar. There are a number of published studies describing the method of measuring head echoes, as reported by Kero et al. (2008). However, although many meteor observers have detected meteor echoes by using VHF beacon waves as well as broadcast radio waves, it has been impossible to observe detailed Doppler shifts from the head echoes of meteors. Under such circumstances, the authors have tried to detect meteor head echoes with Doppler shifts by setting in close proximity a traditional low-cost and low-power transmitter and receiver (which can be obtained by anyone) and observing the results for more than a year. Consequently, we were able to perform a detailed analysis of the Doppler shift for the first time, which was previously considered impossible. This means that any observer who has been involved in amateur radio operations can make observations similar to those obtained with a large-scale pulse meteor radar.

The Eta Aquarids is a meteor stream of the parent comet, Halley (1P/Halley). It reaches its maximum activity on May 5 and 6, almost every year. The right ascension and declination of the radiant are $337\overset{\circ}{.}5$ and $-1\overset{\circ}{.}1$, respectively. In Japan, twilight occurs only for a short duration when the radiant rises above the horizon; therefore, optical observations are difficult. A meteor belonging to the Eta Aquarids meteor stream has an extremely high ground velocity, at $65.5\,\mathrm{km\,s^{-1}}$ (Koseki 2021). Multiple comprehensive studies of the Eta Aquarids shower have been conducted using head echo observations (Chau & Galindo 2008; Schult et al., 2018).

In the present study, we report in detail the Doppler shift of the two meteor head echoes observed in May 2021. Our observations are in agreement with the known physical parameters of the Eta Aquarids meteors.

2. Observation

2.1. Transmitting station

Table 1, Fig. 1 (c), and Fig. 2 (a, b) show the transmitting station of the VHF beacon radio wave and some parameters of the transmitting devices used for this radio observation. The transmitter (TX) was located at Hanakikou observatory (HK; call sign: JA2SDA; location: $34\overset{\circ}{.}819$ N, $137\overset{\circ}{.}344$ E; Fig. 1 (c)). In Japan, the radio band is assigned to ham from $50.0\,\mathrm{MHz}$ to $54.0\,\mathrm{MHz}$. In this experiment, a transmitting frequency of $53.100000\,\mathrm{MHz}$ was used, and the output power was $5.0\,\mathrm{W}$ continuous wave. This VHF beacon radio wave was transmitted for $9\,\mathrm{min}$ and then stopped for $1\,\mathrm{min}$ owing to radio regulations. Subsequently, the call sign was sent out by the Morse signal at the first part of the stopping time of the radio wave. Yaesu FT991AM (Table 1, Fig. 2 (b)), manufactured by Yaesu Radio, was used as the transmitter. The VHF radio wave was transmitted using a 2-element Yagi antenna directed to the zenith (Fig. 2 (a)).

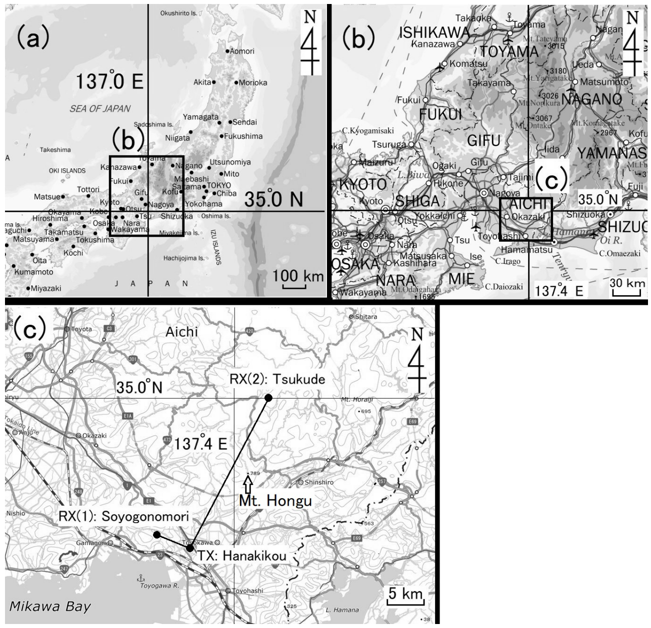

Fig. 1. Locations of the transmitting stations and the two receiving stations. The transmitting station is Hanakikou Observatory (TX), and the receiving stations are Soyogonomori Observatory (RX(1)) and Tsukude Observatory (RX(2)), similar to that presented in (c). (a) a map of central Japan, (b) a map of Tokai region in Honshu, central Japan, and (c) a topographic map of Aichi prefecture including TX and RX(1), RX(2). These maps are published by the Geospatial Information Authority of Japan (GSI).

2.2. Receiving stations

Two receiving stations were set up for the experiments. One receiver (RX(1)) was located at Soyogonomori observatory (SG; location: $34\overset{\circ}{.}838$ N, $137\overset{\circ}{.}291$ E; straight line distance to the transmitting station is 4.53 km; Fig. 1 (c)) and another (RX(2)) at Tsukude Observatory (TK; location: $35\overset{\circ}{.}001$ N, $137\overset{\circ}{.}453$ E; straight line distance to transmitting station is $22.4\,\mathrm{km}$ and Mt. Hongu (height: $789\,\mathrm{m}$) is situated between TX and RX(2); Fig. 1 (c)). The receiving antenna at Soyogonomori observatory (SG) was a 2-element Yagi antenna (Comet CA-52HB), which was directed to the zenith (Fig. 2 (c)). IC-PCR1000 was used as the receiver (Fig. 2 (d)), which is manufactured by ICOM. Similarly, the receiving antenna at Tsukude observatory (TK) was also a 2-element Yagi antenna (Comet CA-52HB), which was directed to the zenith (Fig. 2

Table 1. Parameters of the radio observations of the meteor head echoes analyzed in this study.

	Transmitting station (TX)	Receiving station (1) (RX(1))	Receiving station (2) (RX(2))
Observatory	Hanakikou (HK)	Soyogonomori (SG)	Tsukude (TK)
Location	34°.819 N, 137°.344 E	34°.838 N, 137°.291 E	35°.001 N, 137°.453 E
Transmitter (Output)	Yaesu FT991AM (5 W)	*****************	*****************
Receiver	*****************	ICOM IC-PCR1000	ICOM IC-R7000
Antenna (Elements)	Comet CA-52HB (2-ele.)	Comet CA-52HB (2-ele.)	Comet CA-52HB (2-ele.)
Transmitted or received frequency (Radio wave type)	53.100 MHz	53.099 MHz (USB)	53.099 MHz (USB)
PCM recorder (Format, Sampling freq.)	*****************	TEAC LX-110 (16 bits, 48.0 kHz)	Tascam DR-600MK (16 bits, 44.1 kHz)

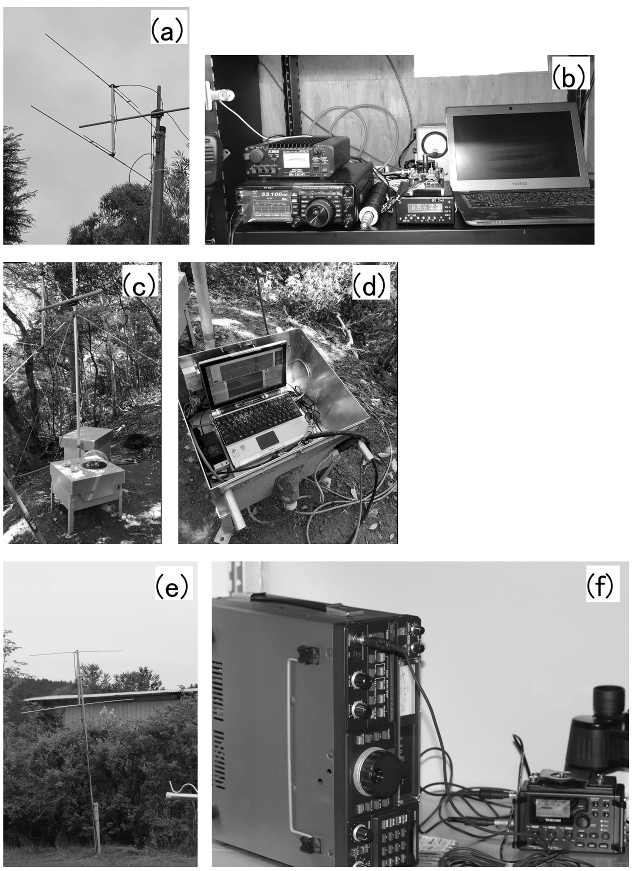

Fig. 2. Images of the observational devices for receiving meteor head echoes in this study. (a): Transmitting 2-elements Yagi antenna (Comet CA-52HB) at TX, HK. (b) Transmitter (Yaesu FT991AM; lower left in photo (b)) at TX, HK. (c): Receiving 2-elements Yagi antenna (Comet CA-52HB) at RX(1) ,SG. (d): Receiver (IC- PCR1000) at RX(1) ,SG. (e): Receiving 2-elements Yagi antenna (Comet CA-52HB) at RX(2) ,TK. (f): Receiver (IC-R7000) at RX(2) , TK.

(e)), and the receiver used was IC-R7000 (Fig. 2 (f)) manufactured by ICOM.

The description provided in this paragraph is common for both the receiving stations. The received wave was a beacon VHF radio wave transmitted from Hanakikou observatory (HK), and USB was the type of received radio wave used. When the frequency of the receiver is synchronized to 53.099000 MHz and the transmitted VHF radio wave of 53.100000 MHz is received by the radio wave type USB, the transmitted signal can be heard as a sound of 1000 Hz. If the Doppler effect of the head echo caused by the meteor velocity component in the line-of-sight direction is obtained, sound with frequency higher than 1000 Hz should be received when a meteor approaches a receiver. In contrast, sound with frequency lower than 1000 Hz should be received when it moves far away. The sound representing VHF radio wave received by the USB radio type was recorded on an IC recorder (Soyogonomori observatory: TEAC LX-110 (16bits, 48.0kHz); Tsukude observatory: Tascam DR-60DMK II (16 bits, 44.1 kHz)). After the observation period, the Doppler effect associated with the head echo of a meteor was verified using frequency analysis with freely available FFT software, Audacity (https://www.audacityteam.org/).

The radio data were recorded for 6 hr a day from 05:00 AM, LT (when the altitude of the radiant of Eta Aquarids is about 40°) to 11:00 AM (the altitude, about 30°) for 5 days from May 3 to 7, 2021.

3. Observational results

Two meteor head echoes with remarkable Doppler shifts were obtained during the 30-hr observation period. Although head echoes of more than 10 meteors were obtained in five days, this study is limited to two typical head echoes in which the Doppler shifts could be reliably observed. The first meteor head echo was obtained at TK at 06:18:00 (LT; LT=UT+9h) on May 4, 2021 (Fig. 3; Fig. 4), and the second was obtained at SG at 08:51:25 LT on May 5, 2021 (Fig. 5). To determine the Doppler effect caused by the head echo, the transmitter and

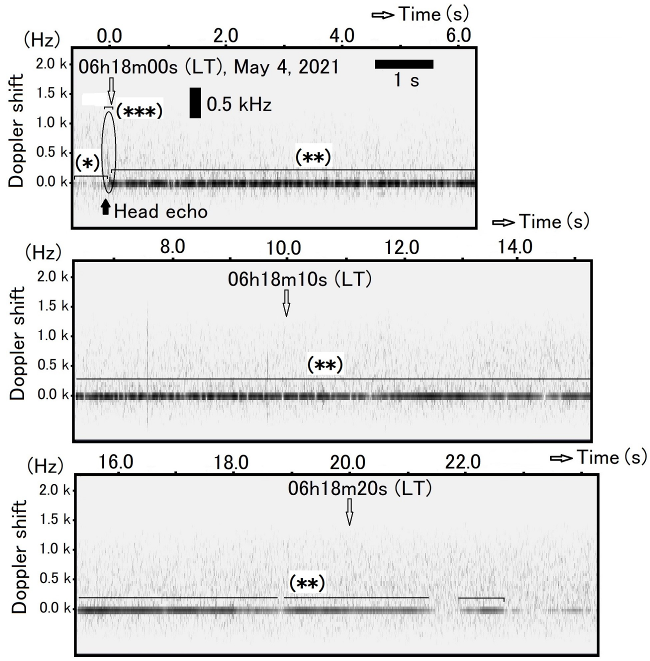

Fig. 3. Frequency analysis images of the total reception of the meteor echoes received at 06:18:00 LT on May 4, 2021 observed at TK by Audacity (https://www.audacityteam.org/).

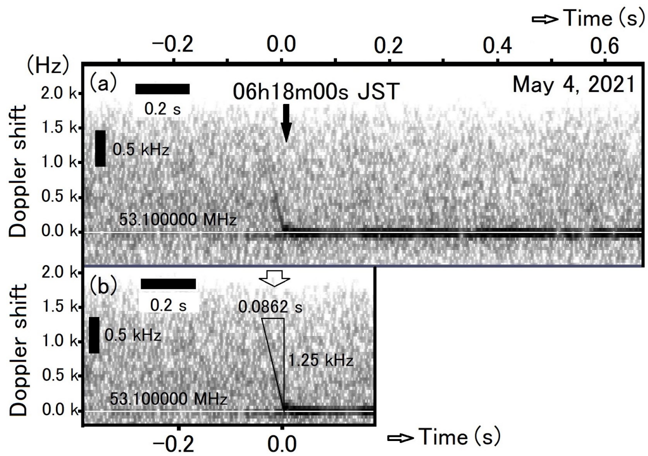

Fig. 4. (a) Frequency analysis image by Audacity, in which the head echo portion of Fig. 3 is enlarged and (b) is a copy of the head echo portion of (a), which is used for the calculation of the Doppler shift.

receiver should be assumed to be at the same location for convenience.

In Figure 3, we show the frequency analysis images acquired continuously by Audacity from the start to the end of reception of the meteor echo received at 06:18:00 LT on May 4, 2021, at TK. The duration of the trail echo from this meteor was approximately 23 s. The (*) symbol in Figure 3 is the transmitted direct wave line of $53.100000\,\mathrm{MHz}$, which shows a slight reception. The distance between the transmitter and receiver is $22.4\ \mathrm{km}$, and Mt. Hongu is situated between them (Fig. 1(c)). Therefore, direct waves are not always received. The dark line extending horizontally from the symbol (**) in Figure 3 is the echo signal from the meteor trail. For the meteor trail echo, the transmitting VHF radio wave is reflected by the

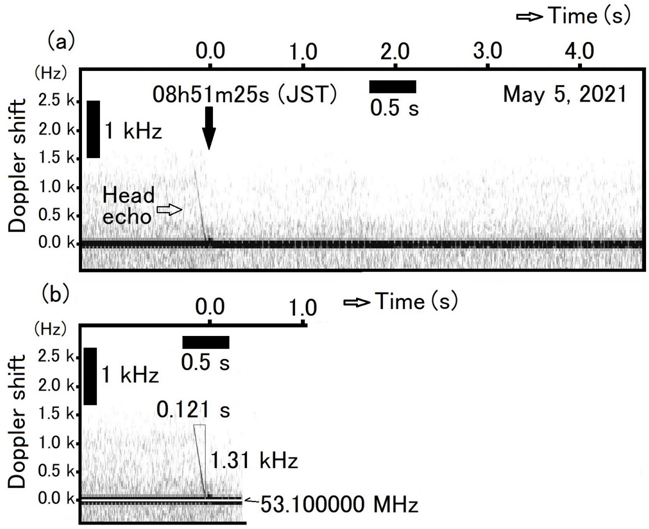

Fig. 5. Frequency analysis images by Audacity (https://www.audacityteam.org/) of the meteor echo received at 08:51:25 LT on May 5, 2021 observed at SG. (a) is the image of the head echo portion. (b) is a copy of the head echo portion of (a), which was used for the calculation of the Doppler shift.

ionized column formed along the meteor path. Given that the ionized components in the E region of the ionosphere move at a rate of approximately 10–100 $\mathrm{m\,s^{-1}}$ (Bourdillon et al. 2005), the Doppler shift should correspond to the velocity of ionized components. However, in such an echo case, the difference in frequency from the direct radio wave is negligible. Therefore, both the signal line indicating the reception of the direct radio wave and the reception of the trail echo almost overlap each other. In the last diagram of Figure 3, it can be seen that the echo signal, after repeatedly rising and falling, eventually disappears.

As shown by the symbol (***) in Figure 3, a pulse (presented as "Head echo") that intersects the head of the trail echo with the tilt orthogonal to the $53.100000\,\mathrm{MHz}$ signal line was found. which is a meteor-head echo. In Figure 4 (a) we present the frequency analysis image acquired by Audacity, in which the head echo portion of Figure 3 has been enlarged. As the authors used backscattered meteor radio observations, the highest reception efficiency of the reflected wave occurred at the point where the meteor path (meteor trail) is orthogonal to the reflected radio wave (f_0 point, Fig. 6) (McKinley 1961). In accordance with the Doppler effect, the frequency of the meteor head echo increased compared to the transmitted frequency when the meteor approached the f_0 point, and it decreased when the meteor moved away. With this particular meteor head echo, the largest Doppler shift was $1.25\,\mathrm{kHz}$ before $0.0862\,\mathrm{s}$ from the f_0 point, where the meteor velocity component in the line-of-sight direction was zero (Fig. 4 (b)).

The frequency analysis images acquired by Audacity of the meteor echo that was received at 08:51:25 LT on May 5, 2021, at SG are shown in Figure 5 (a, b). The distance between the HK (transmitter) and SG (receiver) is only $4.53\,\mathrm{km}$, and the transmitted direct radio wave is always received at SG. Because the frequency of the meteor trail echo is almost the same as that of the transmitted direct radio wave, it is difficult to distinguish them. However, a pulse (presented as "Head echo" in Fig. 5

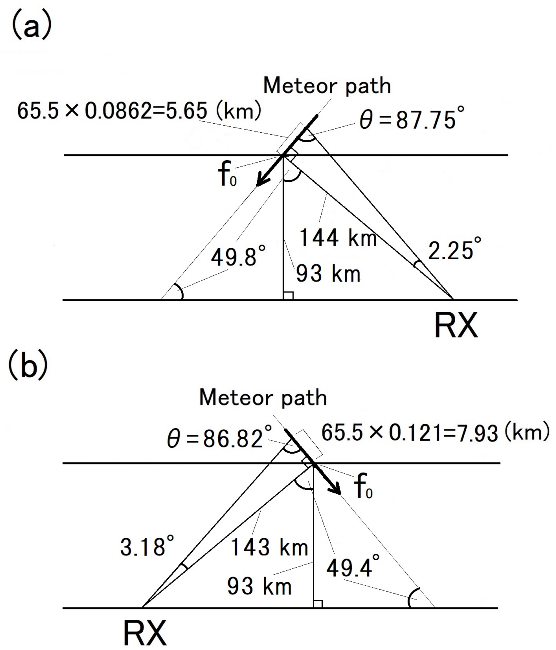

Fig. 6. Relation between the real paths and the Doppler frequencies of two Eta Aquarids meteors whose head echoes were observed. Each value was calculated assuming that both the meteors belong to Eta Aquarids (radiant: right ascension is $337^{\circ}\!.5$, and declination is $-1^{\circ}\!.1$; ground velocity: $65.5\,\mathrm{km\,s^{-1}}$) (a) is the meteor echo received at 06:18:00 LT on May 4, 2021. (Fig. 3, Fig. 4) (b) is the meteor echo received at 08:51:25 LT on May 5, 2021. (Fig. 5)

(a)) that intersects the head of the trail echo with a tilt orthogonally close to the $53.100000\,\mathrm{MHz}$ signal line (similar to Fig. 3 and Fig. 4) was detected, which is also a meteor head echo. With respect to this meteor head echo, the largest Doppler shift was $1.31\,\mathrm{kHz}$ before $0.121\,\mathrm{s}$ from the f_0 point (Fig. 5 (b)).

4. Discussion

In the present study, a continuous wave was transmitted to the zenith to receive a meteor head echo. Assuming that both the transmitter and receiver are located at the same point, the Doppler shift due to moving a meteoroid can be expressed by Eq. (1) (Merrill 2002). If the Doppler frequency f_d is observed, the meteor velocity component in the line-of-sight direction $v\cos\theta$ can be estimated. Furthermore, if the angle θ between the direction of the meteor's movement (meteor path) and the line-of-sight direction of the meteor can be determined, the ground velocity v of the meteor can be determined.

$$f_d = 2v\cos\theta f_t/c, \tag{1}$$

where f_d is the Doppler frequency (Hz), v is the ground velocity ($\mathrm{m\,s^{-1}}$) of the meteor, θ is the angle between the direction of the meteor's movement and the direction of the line of sight of the meteor (degree), f_t is the transmitted frequency (Hz), and c is the velocity of light ($\mathrm{m\,s^{-1}}$).

The ground velocity of the Eta Aquarids meteor is $v = 6.55 \times 10^4\,\mathrm{m\,s^{-1}}$, the transmitted frequency is $f_t = 5.31 \times 10^7$ Hz, and the velocity of light is $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$. The head echo of the meteor received at 06:18:00 LT on May 4, 2021 had a Doppler frequency of $1.25\,\mathrm{kHz}$ at $0.0862\,\mathrm{s}$ before reaching the f_0 point, where the signal line of the transmitted frequency exists (Fig. 4 (b)). If this head echo corresponds to that of the Eta Aquarids meteor, its ground velocity is $65.5\,\mathrm{km\,s^{-1}}$, and the meteor moves $5.65\,\mathrm{km}$ in $0.0862\,\mathrm{s}$ to the f_0 point. The altitude of the radiant of Eta Aquarids meteor is $49^{\circ}\!.8$ at the received time (06:18:00 LT). Assuming that the height of the f_0 point is $93\,\mathrm{km}$, which is the ordinary height of the meteor echoes (McKinley 1961), θ in Eq. (1) was calculated to be $87^{\circ}\!.75$ (Fig. 6 (a)). If these numbers are substituted into Eq. (1),

$$\begin{aligned} f_d &= 2v\cos\theta f_t/c \\ &= 2 \times 6.55 \times 10^4 \times \cos 87^{\circ}\!.75 \\ &\quad \times 5.31 \times 10^7/(3.00 \times 10^8) \\ &= 9.10 \times 10^2\,(\mathrm{Hz}). \end{aligned} \tag{2}$$

This Doppler frequency (9.10×10^2 Hz; Eq. (2)) agrees with the experimental result (1.25×10^3 Hz) obtained from the frequency analysis with 73% accuracy. This result suggests that the received signal is the head echo of the Eta Aquarids meteor. It should also be noted that if the height of the f_0 point is $90\,\mathrm{km}$, the Doppler frequency is calculated as $9.71 \times 10^2\,(\mathrm{Hz})$, and $100\,\mathrm{km}$, $8.50 \times 10^2\,(\mathrm{Hz})$. The experimental and geometric values were more consistent when the height was assumed to be $90\,\mathrm{km}$.

Similarly, the head echo of the meteor received at 08:51:25 LT on May 5, 2021, had a Doppler frequency of $1.31\,\mathrm{kHz}$ at $0.121\,\mathrm{s}$ before reaching the f_0 point (Fig. 5 (b)). If this head echo corresponds to the Eta Aquarids meteor, its ground velocity is $65.5\,\mathrm{km\,s^{-1}}$, which is similar to the former case. It was calculated that the meteor moves $7.93\,\mathrm{km}$ in $0.121\,\mathrm{s}$ to the f_0 point. The altitude of the radiant of the Eta Aquarids meteor was $49.^{\circ}4$ at the received time (08:51:25 LT), and assuming that the height of the f_0 point is $93\,\mathrm{km}$, θ in Eq. (1) was calculated to be $86^{\circ}\!.82$ (Fig. 6 (b)). If these numbers are substituted into Eq. (1),

$$\begin{aligned} f_d &= 2v\cos\theta f_t/c \\ &= 2 \times 6.55 \times 10^4 \times \cos 86^{\circ}\!.82 \\ &\quad \times 5.31 \times 10^7/(3.00 \times 10^8) \\ &= 1.29 \times 10^3\,(\mathrm{Hz}). \end{aligned} \tag{3}$$

This Doppler frequency (1.29×10^3 Hz; Eq. (3)) agrees well with the experimental result (1.31×10^3 Hz) obtained from the frequency analysis with 98% accuracy. The result strongly suggests that the received signal is the head echo of the Eta Aquarids meteor. Similarly, on this meteor, if the height of the f_0 point is $90\,\mathrm{km}$, the Doppler frequency is calculated as 1.33×10^3 Hz, and $100\,\mathrm{km}$, 1.17×10^3 Hz. The experimental and geometric values were the most consistent when the height was assumed to be 90–93 km.

5. Conclusions

By utilizing continuous VHF radio waves (ham radio), two head echoes from the Eta Aquarids meteors were detected at a location in close proximity to the transmitter, Japan, in May 2021. The Doppler shifts of the moving head echoes of the meteors were determined using freely available frequency analysis software. The Doppler shift calculated from the meteor head echo agreed well with the data obtained from the position of the meteor's actual path using the known physical parameters of the Eta Aquarids meteors.

To our knowledge, this is the first detailed report of a Doppler shift of a meteor's head echo using ordinary low-cost and low-power ham radio devices. Our results suggest that it may be possible to estimate some physical parameters of the meteor. Assuming that meteor head echoes can be observed at multiple points simultaneously using this new simple method, the position and ground velocity of a meteor can be estimated. This system will help us find the path of a meteorite during daytime or on cloudy days.

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