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Linear Nonadiabatic Pulsation Models of Ultra-Long-Period Cepheids

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Abstract

Ultra-Long-Period Cepheids (Periods longer than 80 days) are examined using linear nonadiabatic pulsation models. We found models with ultra-long-periods even with Z = 0.02 models, so such a Cepheid may exist in our Galaxy, which has not been discovered yet. We seem to need a survey of the far side of our Galaxy or a survey in the other galaxy with a similar chemical composition to discover such an object. It is also shown that the models with periods longer than 200 days need to be more massive than the Cepheid upper mass limit, inferred from the stellar evolution theory. It is pointed out that the Cepheid's nature of HV 1956 in SMC, which has over-200 days period, needs careful confirmation.

Key words: Stars: individual (HV 1956 in SMC) — stars: oscillations — stars: variables: Cepheids

1. Introduction

Classical Cepheids are radially pulsating population I yellow supergiants. These stars are known to be the standard candles for the extragalactic distance scale, using the Leavitt law, previously called the period-luminosity relation. Their fundamental properties are explained by the stellar pulsation theory. It is found that there are some differences in pulsation properties among the stellar systems of different chemical compositions, including the period-luminosity relation. Therefore, it is interesting to investigate the difference between pulsation models with different chemical composition parameters.

As one of the possible results of the difference in chemical composition, it is known that the longest period of Cepheids in our Galaxy, the Large Magellanic Cloud (LMC), and the Small Magellanic Cloud (SMC) have large differences. While the star with the longest period among the classical Cepheids in our Galaxy is S Vul with $\sim$ 68 days, the Cepheids with periods longer than 100 days have been known in both the LMC and the SMC. Among these stars, the Cepheid with the longest period is the HV 1956 with P = 208.7992166 days [1] However, it is unclear whether the differences in the periods of the Cepheids between the LMC/SMC and our Galaxy are due to the differences in the chemical compositions, or not, because a large portion of the Cepheids in our Galaxy have not been discovered yet due to the heavy extinction to galactic-disk direction. Such Cepheids or Cepheid-like variables with long periods and high luminosities have a special advantage in distance measurement. Grieve, Madore, and Welch (1985) suggested that some bright stars (called by them as "Leavitt Variables") may have the period corresponding to the extension of the Cepheid's period-luminosity relation. Recently, such a possibility with "Ultra Long Period"(ULP) Cepheids has been re-investigated by some authors like Bird, Stanek, and Prieto (2009). The ULP Cepheids are defined by them as the fundamental-mode Cepheids with a pulsation period of longer than 80 days. More recent investigations concerning the ULP Cepheids are summarized in Musella (2022).

There are a few investigations on pulsation models of Cepheids with a long period. Carson and Stothers (1984) studied the Cepheids with periods of longer than 50 days using the dynamic models, but it was before the OPAL opacities appeared. Convective models with periods over 100 days are reported in Fiorentina et al. (2007). However, because only a few models have periods correspond to the ULP Cepheids, it seems difficult to discuss the properties of the ULP Cepheids from the studies of pulsation models. A part of the linear nonadiabatic calculations of the long-period Cepheids are previously reported in Ishida (1998).

In this paper, I would like to examine the existence of the pulsation modes with periods over 100 days and the pulsation stability of the modes using the linear nonadiabatic pulsation models. The evolution model of such a star is not calculated in this paper. I also would like to discuss the Cepheid's nature of the HV 1956. In section 2, the models we constructed are explained. The results are shown in section 3. The discussions are presented in section 4. We summarize in section 5.

2. Models

For the calculation of the linear non-adiabatic pulsation periods, we used the same code used in Ishida (1995), which uses Castor's (1971) type procedure to estimate the periods and the growth rates in the non-adiabatic pulsations using the OPAL opacity tables (Iglesias et al. 1992; Rogers & Iglesias 1992).

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[1] Period of the HV 1956 is from the OGLE Collection of Variable Stars site. https://ogledb.astrouw.edu.pl/ ogle/OCVS/

As usual for radial stellar pulsation in Cepheids, only the stellar envelope is included in the basic equations, since the pulsation amplitude is very small near the stellar core. Accordingly, thermonuclear energy production is neglected and constant luminosity flux is assumed at the inner boundary. The effect of convection is not included in this model, and the envelope is treated as purely radiative. We can calculate pulsation periods and growth rates for radial stellar pulsation modes up to the arbitrary order with this code.

The effective temperature of the models is fixed to 5000 K. The effective temperatures of the long-period Cepheids are lower than those of short-period Cepheids like the double-period Cepheids. Therefore we need to assume the effective temperature values of lower than those values used in the models for the short-period Cepheids (Ishida 2017). Molecular opacities are not included in the opacity tables used.

Considering the result of preliminary calculations, the model mass range is set from 10.0 to 40.0 $M_\odot$ with mass step 2.0 $M_\odot$. We increased the luminosity of the models from 10000.0 $L_\odot$ until the model does not converge with the luminosity step 10000.0 $L_\odot$. For a very high luminosity model, we can calculate the static-model envelope but we cannot obtain the periods of the linear-adiabatic pulsations for no convergence. Because we usually experience the no convergence error in the nonadiabatic part if there is a problem in the numerical calculation, such an error in the adiabatic part seems to indicate that pulsation like Cepheids is not possible in such a region. We will further discuss this no convergence of the code in section 4.

Since the ULP Cepheids are observed in many systems, we calculated models with three different compositions, say, $Z = 0.02$, 0.01, and 0.004.

3. Results

The result for $Z = 0.02$ is shown in figure 1. Constant period lines for 50, 80, 100, 150, 175, 200, and 210 days are indicated by black lines with its fundamental periods indicated aside. Because the ULP Cepheids are defined as Cepheids with a period of longer than 80 days, it is worth noting that the model with a period of longer than 80 days is widely available for the mass range we investigated. The red line indicates no convergence limit. As the mass of the model increases, we can obtain the period for the models with higher luminosities. The longest period we can obtain also increases as the model mass increases. It is worth noting that the period over 200 days becomes possible over 24 $M_\odot$ in $Z = 0.02$ models. The fundamental mode is pulsationally unstable in all the models we calculated.

Figures 2 and 3 are results for $Z = 0.01$ and $Z = 0.004$, respectively. It is shown that the no-convergence line and constant-period lines shift to the higher luminosity direction as the values of Z decrease. However, the longest possible period does not extend to the lower mass region, and we still need more than around 25 $M_\odot$ to obtain the model with a period of over 200 days.

We also summarized in Figure 4, to clarify that the longest period increases as the model mass increases. The longest period reaches 200 days around 24 $M_\odot$ for $Z = 0.02$, and around 27 $M_\odot$ for $Z = 0.01$ and $Z = 0.004$ models.

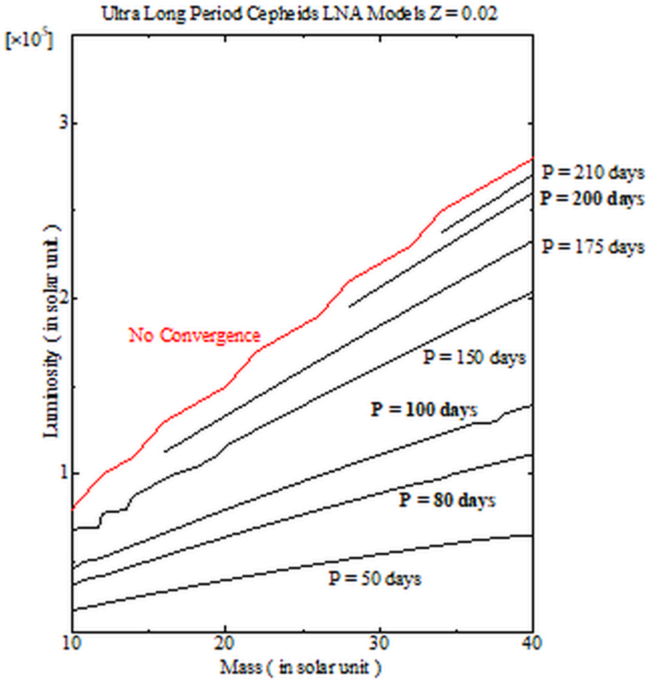

Fig. 1. Constant period plot for $Z = 0.02$. The abscissa is the model mass and the ordinate is the model luminosity both in the solar unit. Black curves are constant-period lines for 50, 80, 100, 150, 175, 200, and 210 days with its periods indicated aside. The red line indicates no convergence limit.

4. Discussions

For evaluating the effect of convection and inclusion of molecular opacities, we tried to use the Radial Stellar Pulsation (RSP) feature of the Modules for Experiments in Stellar Astrophysics (MESA) project, which is described in the MESA paper5(Paxton et al. 2019). We used almost the same parameters as the default values used in the MESA, however, grid points are increased to nearly 500, as our no-convection model also uses nearly 500 zones. We picked up only the results for the linear calculation part of the MESA RSP feature. The most significant difference between the results of MESA RSP and our models is that the MESA models do not converge at the lower luminosity than our radiative models, thus we cannot construct models with a long period. The converged model has periods of about 10% difference and smaller growth rates than the results obtained by the radiative models with the same mass and luminosity, therefore the main results with our models seem not changed by the inclusion of convective energy transport. We have not found a set of model parameters to bring a period consistent with the ULP Cepheids' periods by using the MESA feature. Because the calculated period for the RV Tauri models with the MESA are also reported not to reach

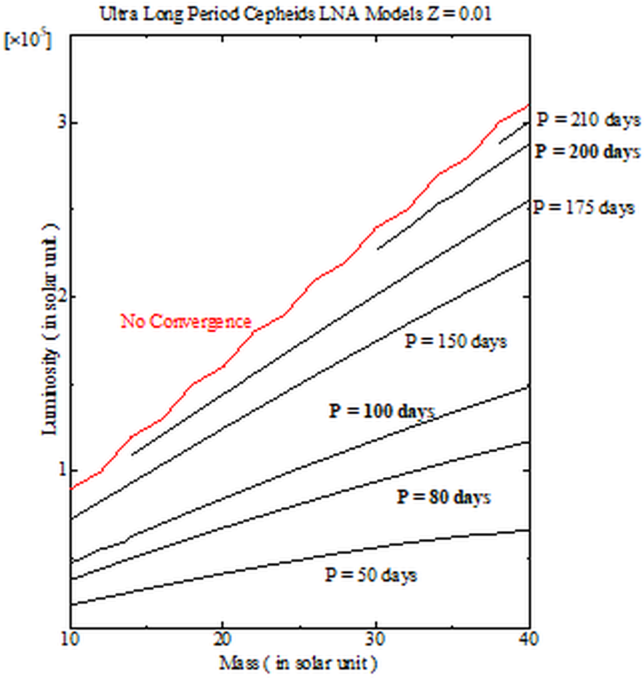

Fig. 2. Constant period plot for Z = 0.01. Notations are similar to figure 1.

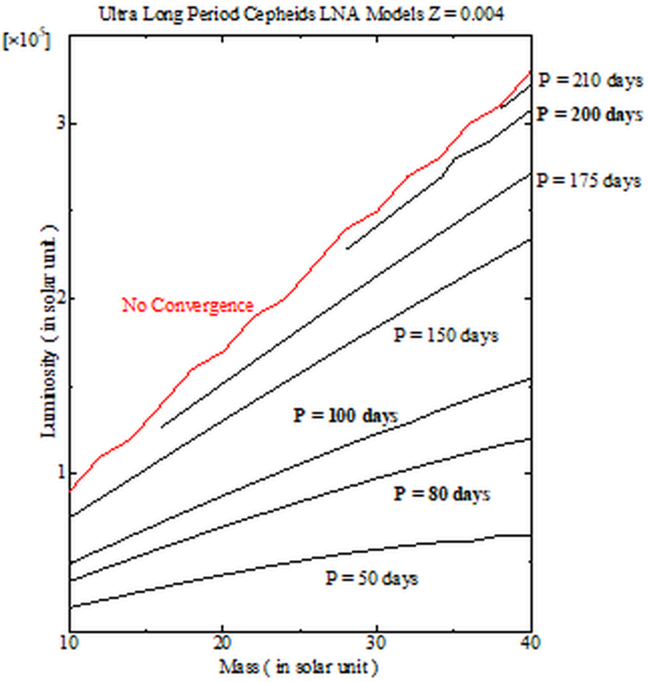

Fig. 3. Constant period plot for Z = 0.004. Notations are similar to figure 1.

a period of longer than 20 days, the MESA RSP feature seems to need modification to extend to the longer period region.

As mentioned in Section 2, our code does not converge at the luminous end of the fixed mass sequence. In most case, the linear-adiabatic part does not converge. We have examined the acoustic cut-off frequency (for example, Unno et al. 1989) profile from the static model and compared it with the period obtained. For the models with a period of longer than 200 days, the period in the linear-adiabatic pulsations is about 230 days while the corresponding period for the maximum of the acoustic cut-off frequency is around 13 days. Therefore, no convergence does not seem from this physical limitation. We have not identified the origin of no convergence for now.

The ULP Cepheids are possible even in Z = 0.02 models, as described in the previous section. Although we have not found Cepheids with a period of longer than 80 days in our Galaxy, we have obtained the envelope model up to 150 days even at 10.0 $M_\odot$. The ULP Cepheid in our Galaxy may be overlooked for the strong distinction to the Galactic disc direction. We seem to need a survey in the far side over the Galactic center to find such an object. It may also be possible to find such an object in other stellar systems with similar compositions like our Galaxy.

We also pointed out that periods over 200 days are possible around 24 $M_\odot$ or more massive regions. Cepheids are usually considered to be observed in the so-called blue-loop phase of the stellar evolution. The upper limit of the initial mass that the model star indicates this evolutionary phase depends

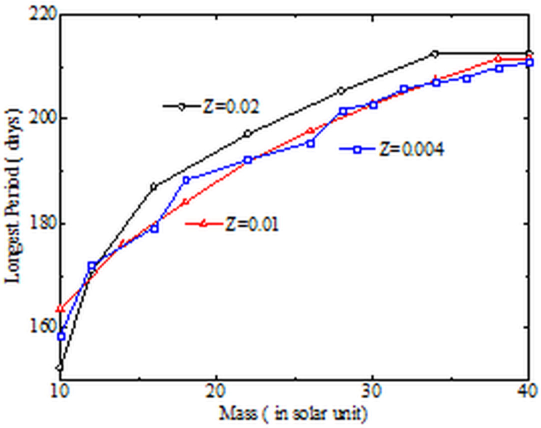

Fig. 4. The longest periods obtained as the model mass changes. The black circles are for Z = 0.02 models. The red triangles and blue squares are for Z = 0.01 and Z = 0.004 models, respectively.

on details included in the evolution calculation. However, for example, Anderson et al. (2014) estimated the upper mass limit for Cepheids as around 12 $M_{\odot}$. Although HV 1956 in SMC has been treated as a classical Cepheid since Shapley and MaKibben Nail (1951), confirmation of its properties as a Cepheid seems to be needed.

5. Concluding Remarks

In this paper, we calculated linear nonadiabatic pulsation periods with the ULP Cepheids models. The ULP Cepheids seem possible even with $Z = 0.02$ models, therefore there is a possibility that such a long-period Cepheid may exist somewhere in our Galaxy. We seem to need a survey of the far side of our Galaxy or a survey of the other galaxies with similar chemical composition.

It is also shown that the models with a period of longer than 200 days need to be more massive than the usual Cepheid upper mass limit. So, although the estimated magnitude may be nearly on the extension of the Cepheid period-luminosity relation, the variables with over 200 days period may be evolutionary different from classical Cepheids. Especially, it is pointed out that HV 1956 in SMC with a period of longer than 200 days, which is usually considered as a classical Cepheid, needs careful confirmation on its nature as a Cepheid.

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